Coordi: A Virtual Reality Application for Reasoning about Mathematics in Three Dimensions

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ABSTRACT

The goal of our research has been to create software that extends the benefits of virtual reality (VR) to mathematics education. We report on the design and evaluation of a VR application meant to support students' reasoning about objects in three-dimensional (3D) coordinate systems and to explore the possibilities of the application for mathematics education in high school classrooms.

CHI'19 Extended Abstracts, May 4–9, 2019, Glasgow, Scotland UK

ACM ISBN 978-1-4503-5971-9/19/05.
https://doi.org/10.1145/3290607.3312931

KEYWORDS
virtual reality; 3D graphing; 3D vector mathematics; mathematical reasoning; mathematics education; embodied cognition

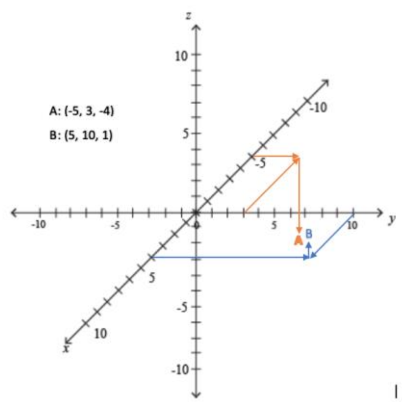

Figure 1: Plotting Points A and B.

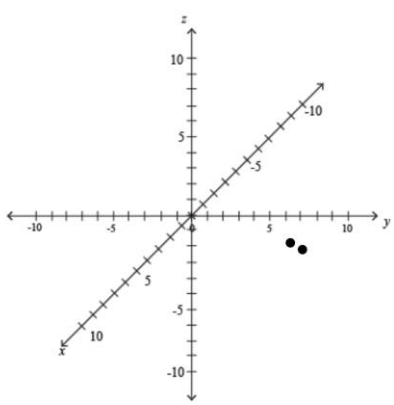

Figure 2: Result of Plotting Points A and B.

1 INTRODUCTION

An introductory calculus student sits at their desk staring at a piece of graph paper, pencil in hand. They are learning about the three-dimensional coordinate system and have been asked to plot two points: A (-5, 3, -4) and B (5, 10, 1). Following the instructions in their textbook, they begin plotting A by tracing along each of the axes until they reach the desired position. They do the same for B and mark each point on their paper. The student has just plotted two points in two vastly different locations. In spatial terms, A is "down" (below the x-y plane) and "away" from the viewer (behind the z-y plane), while B is "up" (above the x-y plane) and "towards" the viewer (in front of the z-y plane), as illustrated in Figure 1. A plot of the points on the traditional two-dimensional representation of the three-dimensional coordinate system is shown in Figure 2. The relationship between the points is not clear; they appear to be next to each other despite being in separate regions of space.

Imagine an alternative scenario: A student stands in the middle of the classroom, walking around holding motion-tracked controllers. From inside a headset, they see themselves picking up and moving points in the 3D space around them. They can plot points by placing them in the air around a floating 3D coordinate system. They can move around the points to get a sense for their relative positions from multiple perspectives. Virtual Reality (VR) systems like the Oculus Rift [1] and HTC Vive [2] use tracked headsets and controllers to create immersive virtual experiences. The potential applications of VR for classroom learning are just beginning to be explored [3]. Much of the research on VR designed for education has focused on medical and engineering material [4] and has found that immersive software is more beneficial than both non-immersive software (such as Cabri 3D [5]) and traditional educational methods (such as lectures and textbook instruction). Our work extends this research by exploring the potential benefits of VR for mathematics learning, in particular, benefits for learning about vectors.

Central to both physics and engineering, vector math is notoriously challenging for students to master [6-10]. Some work has already begun to address the need for alternatives to traditional paper and pencil approaches. Karnam, Agrawal, and Chandrasekharan [11] created the "Touchy-Feely Vector" (TFV) application, a 3D modeling desktop application which allows users to create vectors by clicking and dragging on a computer screen. As the student constructs a vector, TFV displays its numerical representation alongside it. By creating a digital representation that depicts the relationship between these representations, TFV was able to improve students' understanding of an abstract mathematical concept. Though an improvement over traditional pencil-and-paper methods, the application is still limited. If the student from our introduction tried to plot their two points using this method, they would still be forced to do so in a two-dimensional world, and the spatial relationship between the points would remain obscure. This is an inherent limitation of desktop-based programs that attempt to depict three-dimensional objects on two-dimensional screens.

Figure 3: User wearing an Oculus Rift virtual reality headset.

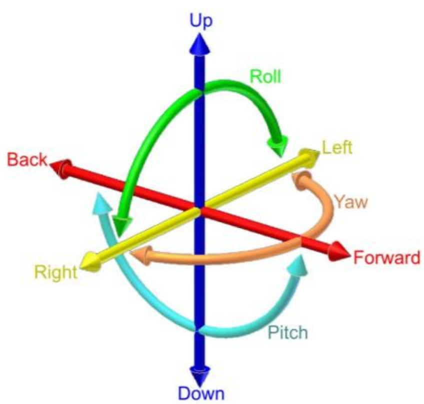

Figure 4: Six degrees of freedom.

2 THEORETICAL ORIENTATION

Thinking about objects in three-dimensional space can be challenging when the representational infrastructure is two-dimensional. Wilensky and Papert [12] argued that representational systems have different affordances and limitations for learning. The introduction of Hindu-Arabic numerals, for example, made mathematical procedures such as multiplication and division straightforward; Roman Numerals had made such procedures anything but approachable. They called the new, more accessible representational form a "restructuration." We argue that VR can be an effective tool for restructurating spatial concepts in mathematics (such as the trajectories of bodies through space, or the shapes of more complex geometrical objects). VR applications that allow students to plot points in a 3D virtual world (as in the opening scenario) can make graphing points in three dimensions more intuitive by grounding the activity in students' physical experience. Theories of embodied cognition suggest that cognition, even when focused on seemingly abstract concepts, is intrinsically linked to the body and the kinaesthetic senses [13]. According to Abrahamson and Lindgren [14], "Learning environments for math and science can be made more effective if they are designed to tap into bodily know-how." The design of technologically-enhanced learning environments has previously been restricted by the limited nature of non-immersive software. With its capacity for creating fully-immersive experiences, VR software is poised to take advantage of principles of embodied cognition and create novel representational infrastructures for domains that require three-dimensional reasoning, including calculus, vector mathematics, and geometry.

3 METHODS

3.1 Purpose

The purpose of our study was to evaluate and refine the design of a VR application meant to help students plot points, draw and manipulate graphs, vectors, and objects, and to reason in 3D space.

3.2 Design

Our application was created using Unity 3D, and its scripting was written in C#. It was designed for the Oculus Rift, a PC-based VR system shown in Figure 3. Of critical importance to the design of the application was the capability of the system to provide tracking with six degrees of freedom. As illustrated in Figure 4, six-degree-of-freedom tracking allows both the rotation and position of a user's headset and controllers to be mirrored in the virtual space. This enables the user to move naturally within a virtual space. The Rift provides six-degree-of-freedom tracking for both the headset and controllers, which enabled the creation of an Embodied User Interface.

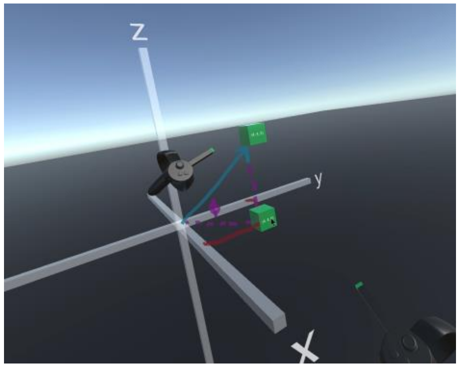

Figure 5: Vector projection, front view.

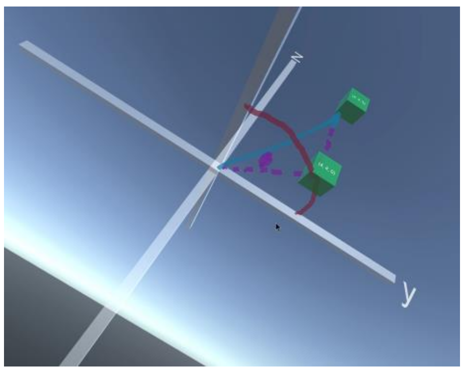

Figure 6: Vector projection, bottom view.

3.3 Application Specifications

The product of our work was an immersive VR application that allows users to plot points and draw in a 3D virtual world. Users are able to move themselves and virtual objects with a full six degrees of freedom. They are able to walk around and change their point of view naturally; they can grab, move, and place objects in the virtual space with the same range of motion they would use to interact with objects in the real world. The user has the ability to plot points (represented by small cubes) by picking them up and placing them at a desired position in space, as indicated by a large 3D axis floating in the middle of the world. Each point has a label on its front. These labels update in real time to display the point's current position in terms of its Cartesian coordinates as it is moved through space. The user also has the ability to draw in three dimensions by holding down a button and moving their controller through the air in front of them. The effect is similar to that of a skywriting airplane, but on a much smaller scale and mediated by a writing utensil in the user's hand. This gives the user the ability to draw complex sketches of graphs, to perform calculations, and to record their work in the space around them. They are also able to place and manipulate planes and use a virtual straightedge to assist their drawing of graphs, vectors, and geometric objects. Figures 5 and 6 depict a mathematical object drawn using the application, and the ability to view it from different angles.

3.4 Evaluation

Our evaluation consisted of two phases, focused on collecting quantitative and qualitative data. We selected participants who had not yet studied mathematics that involved 3D graphing or reasoning. For the quantitative evaluation, we recruited participants from the Northwestern University Center for Talent Development summer program. These participants were academically-oriented high school students (ages 14-18). We randomly divided these 16 participants into two groups of equal size. Each was asked to solve the same set of five questions, which required graphing or reasoning in 3D space (e.g. "Sketch the surface represented by the equation x + y = 2). One group used our VR application, the other used a traditional paper-and-pencil method. Participants' responses were scored on a 10-point scale; 5 points were awarded for drawing a correct model, another 5 for providing the correct answer.

For the qualitative evaluation, we recruited 10 participants from the Northwestern University undergraduate population. Each was given a set of six questions which similarly required graphing or reasoning in 3D space. They were asked to solve three of these using the application we created, and the other three using the "traditional" pencil and paper-based method. We then asked participants to compare their experience using the two approaches and respond to a brief survey identifying the benefits and drawbacks of each method (e.g. "Did you find the virtual reality tool easy to use? If not, what aspects did you find most challenging?").

4 RESULTS

The results of our evaluations suggest that our VR application provides a means for enhancing math learning outcomes. Our quantitative analysis produced optimistic results. The mean score (out of 50 possible points) of participants using the VR tool was 38.125 (SD = 6.74), while those using a traditional paper-and-pencil method to solve the same problems achieved a mean score of 31 (SD = 6.16). While our sample size remains too small to provide evidence for a statistically significant separation of mean scores for the two groups, these initial findings are promising. Our qualitative analysis provided evidence that our VR software enhanced participants' ability to visualize mathematical concepts in three dimensions. The survey respondents noted that the software helped them "picture the coordinate space/objects" and to better "visualize and draw 3-D structures." Evaluations of the VR tool indicated a spatial understanding of the material which was absent from evaluations of the traditional paper-and-pencil method. Participants did note, however, that their greater familiarity with the traditional method provided a certain comfort which the VR approach lacked.

5 CONCLUSION

VR applications like ours can help users represent and reason about mathematical objects in three dimensions. To further our investigation into these potential benefits, we wish to extend our software to support multiple users in collaborative graphing and problem-solving. We also believe the design and evaluation of high school mathematics units focused on drawing, transforming, and manipulating vectors and objects in 3D space will provide greater insights into the ways the application might support student learning.

More broadly, our work provides a tangible example of the benefits of Embodied User Interfaces for education. Human knowledge is intrinsically linked to kinesthetic experiences. Virtual reality allows for the creation of human-computer interactions which tap into embodied cognitive processes, and this presents a number of educational benefits.

First, VR software has the potential to build upon the benefits presented by non-immersive software. In their meta-analysis, Merchant, Goetz, Cifuentes, Keeney-Kennicutt, & Davis [4] found that non-immersive educational software benefit users' learning outcomes. The powerful aspects of this software are, in the very least, replicable in VR.

Second, VR software can be designed to take advantage of principles of embodied cognition. Our software is designed to support interactions between the user and the world that give the user the tangible experience of constructing concrete vectors and objects in the 3D world around them. Hardware plays a crucial role in allowing this; the use of 6-degree-of-freedom headsets and controllers allows the tracking of rotational orientation as well as position, essential elements when attempting to mimic natural movements.

Finally, the novel media format of VR allows for the creation of more intuitive conceptual representations. This supports the creation of representations that more clearly and intuitively display abstract concepts, such as 3D vectors. In mathematics, many concepts (like 3D graphing) are made difficult to understand by the representations we give them. VR provides a new tool for creating more accessible representations of complex mathematical concepts. Our study highlights the educational benefits such new representations (or "restructurations") can provide.

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